DOCUMENT

UFD0016 Jean Petitot

To Remake the 'Timaeus'

In this introduction to Albert Lautman's mathematical philosophy, Jean Petitot reaffirms the importance of a neglected thinker, and outlines Lautman's extraordinary rearticulation of platonism, realism, dialectics, and the history and phenomenology of mathematical creativity

Although very little studied, and surprisingly little known—this undoubtedly being connected to his tragic premature death and the eclipse of philosophy of science in the post-war years—Albert Lautman has nevertheless already been labelled: as a Platonist,[2] as some would have him, despite his exceptional mathematical learning and his close personal ties with Jean Cavaillès, Claude Chevalley, and Jacques Herbrand; as the obsolete remnant of an archaic (Brunschvicgean) idealism, and, for this reason, not 'truly' modern. Mario Castellana repeatedly emphasizes this in his excellent review of the *Essay on the Unity of Mathematics*[3] published in *Il Protagora* where, having summed up Lautman's text, he concludes, as a well-advised connoisseur of French epistemology, that whereas Cavaillès's mathematical philosophy is free 'of all Brunschvicgian philosophico-speculative influence, this influence is still present in Lautman', and that this Platonism 'is to blame for the limited success of Lautman's thought outside of specialist circles, unlike that of Cavaillès'. Whereas Cavaillès (like Bachelard after him) sought to free reflection on mathematics and the sciences from all philosophical legislation claiming to impose some theory of cognition upon it, on the contrary Lautman, interpreting Hilbertian axiomatic structuralism in terms of a Platonic and Hegelian dialectic of the concept, tried to develop an authentic mathematical *philosophy* and, in doing so, failed to take proper account of the tendency toward the *autonomisation* of the sciences.

Lautman represents, without exaggeration, one of the most inspired philosophers of the twentieth century

This diagnosis does indeed accurately reflect what the few rare readers of Lautman tend to take from his work; yet a deeper reading leads us to revise it.

To state it from the outset, in our opinion Lautman represents, without exaggeration, one of the most inspired philosophers of the twentieth century. His theses are of a real importance, and if just a fraction of the reflection dedicated to another philosopher, to whom he is of comparable stature but of opposing ideas—namely, Wittgenstein—had been directed toward Lautman instead, he would without doubt have become one of the most glorious figures of our modernity. The following few remarks on his work aim to help right this injustice.

1. A Philosopher-Mathematician

With Lautman we are in the presence of a philosopher of mathematics who is actually talking about

1. First published as 'Refaire le « Timée ». Introduction à la philosophie mathématique d'Albert Lautman', *Rev. Hist. Sci.* 1987, XL/1. Text © Jean Petitot.

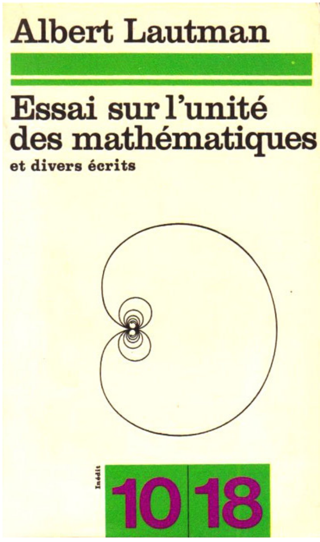
2. Or even 'neo-platonist', according to J. Ullmo, *La Pensée scientifique moderne* (Paris: Flammarion, 1969).

3. M. Castellana, 'La Philosophie mathématique chez Albert Lautman', Il Protagora 115 (1978): 12–24. This is one of the (too) rare recent texts on Lautman, and constitutes a good introduction to his philosophy.

mathematics *and* about philosophy—something that is, it must be said, exceptional if not unique. He does not think that philosophy of mathematics can be reduced to a secondary epistemological commentary on foundational logical problematics, nor to historical nor a fortiori psycho-sociological inquiries, nor to reflections on marginal currents such as intuitionism. Jean Dieudonné quite rightly insists on this in his preface to the *Essay on the Unity of Mathematics*:

> Contemporary philosophers who are interested in mathematics usually concern themselves with its origins, with its relations to logic or the 'problems of foundation' [...] Very few of them seek to construct an idea of the major tendencies of the mathematics of their times, and of what it is that, more or less consciously, drives contemporary mathematicians in their work. Albert Lautman, on the contrary, seems always to have been fascinated by these questions. [...] He developed views on the mathematics of the 20s and 30s more wide-ranging and precise than most mathematicians of his generation, who often were very narrowly specialised. [...] [He] foresaw extraordinary developments in mathematics whose advent his fate would deprive him of the opportunity to see, but which would have filled him with enthusiasm....[4]

This is an important point. Contemporary mathematical philosophy, as denounced by Dieudonné in his polemical article, is a philosophy of the logicist and/or intuitionist persuasion which, with its predeliction for languages, symbolico-categorical structures, and their grammars, rather than 'real' objects[5] and their structures, boasts the curious privilege of miscognizing what is essential in the creative activity of mathematicians. Recall his outburst about Russell's 'stupidity' in wanting to make mathematics a part of logic: 'an enterprise that is as absurd as saying that the works of Shakespeare or Goethe are a part of grammar!'[6] Now, surely the least one can demand and expect of an authentic gnoseological reflection upon mathematics is that it should



develop, in Catherine Chevalley's words, 'a philosophy of sciences intrinsic to theories', a philosophy founded on the genius, the richness, and the novelty of the fundamental discoveries which are to science what the works of a Goethe or a Shakespeare are to literature. 'This is what Lautman understood, and he took up the whole body of the mathematics of his times in order to make it an object of philosophical study. Unfortunately, I have the impression that in this respect he has scarcely any successors.'[7]

His 'effort' was to consist in 'making metaphysics depend not upon the "pathic", but upon the "mathematic"'

But although a mathematician, Lautman is also a real philosopher. Unlike almost all scientists (and equally, alas, most contemporary philosophers), he neither ignored nor disdained either Platonism, metaphysics, or the transcendental. He did not, like others, seek to disqualify the pure thought of being, but on the contrary sought to realise a new dialectical moment of this thought, via the history of pure mathematics. As he confided in an unpublished letter of July 18, 1938 to Henri Gouhier (a specialist in the period of Descartes, Malebranche, and Comte), his 'effort' was to consist in 'making metaphysics depend not upon the "pathic", but upon the "mathematic"'.

4. J. Dieudonné, Introduction to A. Lautman, *Essai sur l'unité des mathématiques et divers écrits* (Paris: UGE, 1977), 15, 19.

5. 'Real' in the sense of idealities.

6. J. Dieudonné, 'Bourbaki et la philosophie des mathématiques', in *Un siècle dans la philosophie des mathématiques* (Brussels: Office international de Librairie, 1981), 178.

7. Ibid., 186.

2. The Elements of Lautman's Philosophy

(a) The Positing of Governing Ideas [idées directrices]

Albert Lautman's central idea is that an *intellectual intuition* is at work in mathematics, and that, as the theories of the latter develop *historically*, they realise an authentic *dialectic of the concept* (in a 'Platonic' but also quasi-Hegelian sense), developing their *unity*, unveiling their *real* and determining their philosophical *value*. It is in virtue of this 'abstract and superior' dialectic[8] that, for Lautman, 'the rapprochement of metaphysics and mathematics is not contingent but necessary'.[9]

Following Dedekind, Cantor, and Hilbert, Lautman thus accorded an ontological import to creative freedom in mathematics. As Maurice Loi notes, one of the characteristics of modern mathematics is that in it, 'mathematical entities are introduced by veritable creative definitions which are no longer the description of an empirical given.'[10]

'In thus liberating mathematics from the task of describing an intuitive and given domain, a veritable revolution is heralded, whose scientific and philosophical consequences are not always duly appreciated'.[11] For, as Loi adds, 'such a conception of mathematical science [...] poses in new terms the problem of its relation to the real, of objectivity and subjectivity. Modern empiricists are happy to oppose science to subjectivism and voluntarism. But objectivity is never a given; it is a quest whose extreme points are axiomatics and formal mathematics.'[12]

Lautman prophetically understood that the *structural* (Hilbertian) conception of mathematics, far from leading to a conventionalist nominalism and relativism, on the contrary led to a new, sophisticated (in fact, transcendental) form of realism. But in emphasising the autonomy, unity, the philosophical

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value, the ideal 'real', the relation to empirical reality, and the ontological import of mathematics, he distanced himself from the dominant tendencies of the epistemology of his times.

Formalist and structural in the Hilbertian sense, his conception was particularly opposed to nominalist, relativist, and sceptical interpretations of conventionalism. This is a particularly delicate point. On the plane of the cultural history of ideas, it is true that Lautman is, along with Cavaillès, one of those who militantly introduced German axiomatics into a French context dominated by the 'intuitionisms' and 'instrumentalisms' of Poincaré, Borel, Baire, and Lebesgue. While remaining faithful to certain aspects of the idealism of his *maître*, runschvicg, he played a determining role in the formation of what was to become the Bourbakian spirit. But on a philosophical plane, the question of conventionalism far surpassed these differences in tendency and conflicts between 'schools'. All the more given that Poincaré's conventionalism—which, in spite of what has been said, was unrelated to any scepticism or relativism—treated of the relations between mathematics and the eidetico-constitutive a priori of regional physical ontology, and could therefore be interpreted in Kantian terms. To make such an interpretation one need only begin again from the concept of the transcendental aesthetic. As we know, the latter is the object of a twofold *exposition*: the metaphysical exposition exhibiting space and time as forms of sensible intuition, and the transcendental exposition exhibiting them in their relation to mathematics. It is through the latter that the forms of intuition, to which of course phenomena must a priori conform, become *methods of mathematical determination*. To mark the difference between them, Kant introduces the concept of *formal intuition* —that is to say, a pure intuition determined as object. The space of geometry is more than a phenomenological continuum, more than a form of *intuition*. As a conceptually-determined formal intuition, it is also a form of *understanding*. But Kant

8. Lautman, *Essai sur l'unité des mathématiques et divers écrits*, 204 (this volume is a republication of the works published by Hermann between 1937 and 1939, and posthumously in 1946.

9. M. Loi, Preface to Lautman, *Essai sur l'unité des mathématiques*, 9.

10. Ibid.

11. Ibid.

12. Ibid.

thinks there is only one geometrical determination of phenomenological space (the unicity of Euclidean geometry). The development of non-Euclidean geometries proved him wrong on this point, and numerous later philosophers have used this as a justification for the wholesale liquidation of the synthetic a priori in the sciences. Conventionalism proposes an alternative to this radical antitheoretical conclusion.[13] For the problem is in fact that of the underdetermination of the form of intuition by formal intuition. To become geometrical, the a priori of sensible space (representational space) must be *idealised*. Now, although empirically constrained, this process of idealisation is empirically (and experimentally) *undecidable*. It concerns an a priori formal faculty of intellectual abstraction that is autonomous in relation to sensible experience. Given this underdetermination, and on the other hand this autonomy, some criteria must be available in order to choose how the determination will be carried out—for example, that of 'convenience'. If intuitive space as phenomenological continuum (as 'amorphous' form, as Poincaré said) does indeed preexist experience, then, and is a condition of possibility for its organisation, the same does not go for geometrical space. Its geometry is conventional, neither empirical nor a priori necessary. Yet it is nonetheless empirically conditioned and theoretically *constitutive*, a priori objectively determining for physics.

Although rationalist, Lautman's conception is also, and above all, opposed to the logicism of the Vienna Circle, which for him represents a 'resignation that the philosophy of science must not accept'.[14] In reprising the dogmatic (i.e. pre-critical) face-off 'between rational knowledge and intuitive experience, between *Erkennen* and *Erleben*'[15], logicism 'suppresses the links between thought and the real'.[16] Its antitheoretical nominalism prevents it from philosophically elucidating the gnoseological fact that the universe is mathematical intelligible. In all of his writings, Lautman repeatedly returns to the philosophical poverty and miscognizing of the mathematical real typical of empiricism and logical positivism, which 'separate, as with an axe, mathematics and reality'.[17]

(b) Structuralism, the Real, the Dialectical

Lautman's conception of mathematics—a structuralist conception—thus reclaims the Hilbertian axiomatic, a non-constructivist axiomatic which

> replaces the method of genetic definitions with that of axiomatic definitions, and, far from seeking to reconstruct all of mathematics on the basis of logic, on the contrary, in passing from logic to arithmetic and from arithmetic to analysis, introduces new variables and new axioms which in each case broaden the domain of consequences.[18]

Between the psychology of the mathematician and logical deduction, there must be a place for an intrinsic characterisation of the real

Born 'of the feeling that in the development of mathematics a reality asserts itself whose recognition and description is the function of mathematical philosophy',[19] taking up Brunschvicg's 'idea that the objectivity of mathematics [is] the work of the intelligence, in its effort to triumph over the resistances that the matter upon which it works opposes to it',[20] and positing that 'between the psychology of the mathematician and logical deduction, there must be a place for an intrinsic characterisation of the real',[21] it could even be called, more precisely, *both axiomatic-structural and dynamic*. This synthesis of a real that 'participates both in the movement of intelligence and logical rigour, without being conflated with one or the other'[22] is what Lautman aims for. It obviously does not come easily, since

> [t]he structural conception and the dynamic conception of mathematics seem at first opposed to each other: the former tends to consider a

13. For a brief presentation of conventionalism, see for example P. Février, 'La philosophie mathématique de Poincaré, in *Un siècle dans la philosophie des mathématiques*.

14. Lautman, *Essai sur l'unité des mathématiques*, 285

15. Ibid.

16. Ibid.

17. Ibid., 145.

18. Ibid., 26.

19. Ibid., 23.

20. Ibid., 25.

21. Ibid., 26 [emphasis ours].

22. Ibid., 26.

> mathematical theory as a completed whole, independent of time; while on the contrary the latter does not separate it from the temporal stages of its development; for the first, theories are like qualitative beings, distinct from each other, whereas the second sees in each theory an infinite power to expand itself beyond its limits, and to link itself with others, thus affirming the unity of intellection.[23]

It is qua structural, in the autonomous and historical movement of the elaboration of its theories, that mathematics realises dialectical ideas and, through them, appears to

> recount, amidst those constructions in which the mathematician is interested, another, more hidden story, one made for philosophy.[24]

> Partial results, rapprochements aborted halfway, attempts that still resemble gropings, organise themselves under the unity of a common theme, and in their movement allow us to perceive a liaison between certain abstract ideas being sketched out—which we propose to call dialectics.[25]

> We do not understand by 'Ideas' models of which mathematical beings are just copies, but, in the true Platonic sense of the word, the schemas of structure according to which effective theories are organised.[26]

As in every dialectic, these schemas of structure establish specific liaisons between contrary notions: local/global, intrinsic/extrinsic, essence/existence, continuous/discontinuous, finite/infinite, algebra/analysis, etc. Alongside facts, beings, and mathematical theories, they constitute a fourth layer of the mathematical real.

> The nature of mathematical reality can be defined from four different points of view: the real is sometimes constituted of mathematical facts, sometimes mathematical beings, sometimes theories and sometimes the Ideas which govern those theories. Far from being opposed to each other, these four conceptions are naturally integrated with each other: the facts consist in the discovery of new beings, which are organised into theories, and the movement of those theories incarnates the schema of liaisons between certain Ideas.[27]

This said, the key to Lautmannian idealism is that, if mathematics is governed by a Dialectics of the Concept (and if, by the same token, mathematics is interdependent with the history of culture), this dialectics nevertheless *only exists qua mathematically realised and historicised*; in other words, 'the *comprehension* of the Ideas of this Dialectics *necessarily extends into the genesis of effective mathematical theories*'.[28] Lautman insists a great deal on this point, which alone suffices to distinguish his conception from a naive subjective idealism.

> In seeking to determine the nature of mathematical reality, we have shown [...] that mathematical theories can be interpreted as a preferred medium destined to embody an ideal dialectic. This dialectic seems to be constituted principally by couplets of contraries, and the Ideas of this dialectic present themselves in each case as the problem of liaisons to be established between opposed notions. The determination of these liaisons can only take place in domains wherein the dialectic is incarnated.[29]

One might say that, according to Lautman, in a certain sense the dialectics of the concept and the mathematics which embody it entertain a relation of 'internal exclusion'. In virtue of the 'intimate union' and the 'complete independence' correlating them (and this without any paradox arising), 'mathematical theories develop through their own force, in a close reciprocal interdependence and without any reference to the Ideas that their movement approaches'.[30]

(c) Comprehension and Genesis

As Gilles Deleuze has emphasised, this leads quite

23. Ibid., 27.
24. Ibid., 28.
25. Ibid.
26. Ibid., 204.
27. Ibid., 135.
28. Ibid., 203 [emphasis ours].
29. Ibid., 253 [emphasis ours].
30. Ibid., 134.

naturally to a philosophy of problems.[31] The dialectical Ideas are purely problematic (not determinative of an object), and as such are constitutively incomplete ('discompleted' of that which would bring them into existence). They 'constitute only a problematic relative to actual situations of the existent' and thus manifest 'an essential insufficiency'[32] And this is why

> [t]he logical schemas (the ideas at work in theories) are not anterior to their realisation within a theory; what is lacking, in what we call [...] the extra-mathematical intuition of the urgency of a logical problem, is a matter to grapple with so that the idea of possible relations can give birth to a schema of veritable relations.[33]

This is also why, in Lautman, mathematical philosophy

> is not so much a matter of rediscovering a logical problem of classical metaphysics within a mathematical theory, as one of globally apprehending the structure of this theory in order to isolate the logical problem which is at once defined and resolved by the very existence of the theory.[34]

> The fundamental consequence of this is that the constitution of new logical schemas and of the unveiling of Ideas depends on the progress of mathematics itself.[35]

Lautman identifies the relation between incomplete problematic Ideas and their specific realisations with the passage from essence to existence

Lautman identifies the relation between incomplete problematic Ideas and their specific realisations with the passage from essence to existence. Drawing the most extreme consequences from the *ideality* of mathematical entities, and from the nature of thinking as thinking of being, Lautman makes the *comprehension* of Ideas the source of the genesis of real theories. By 'incarnating themselves' in actual, effective theories, Ideas are realised within these theories as their foundation and thus—dialectically—as the *cause of their existence*.

> Thought necessarily engages in the elaboration of a mathematical theory as soon as it seeks to resolve [...] a problem susceptible to being posed in a purely dialectical fashion, but the examples need not necessarily be taken from any particular domain, and in this sense, the diverse theories in which the same Idea is incarnated each find in it the reason of their structure and the cause of their existence, their principle and their origin.[36]

It is essential to note here Lautman's reference—an explicit one—to Heidegger. The passage from essence to existence, 'the extension of an analysis of essence into the genesis of notions relative to the existent'[37]—and thus the transformation of the comprehension of a *sense* into the genesis of *objects*—reprises the Heideggerian *ontological difference* between Being and beings. Lautman insists a great deal upon this, in particular in his *New Researches*.

> As in Heidegger's philosophy, one can see in the philosophy of mathematics, such as we conceive it, the rational activity of foundation transformed into the genesis of notions relating to the real.[38]

Thus we come back to the transcendental problematic of ontology as constitution of objectivities. For Lautman—and this poses serious problems for interpretation, to which we shall return—the Dialectic of Ideas is *ontologically constitutive*. In other words, in his work it assumes the function of a *historicised* categorial Analytic. We can draw a parallel between the correlations *Ideas-theories* and *ontological-ontic* because

> The constitution of the being of the existent, on the ontological plane, is inseparable from the determination, on the ontic plane, of the factual

31. See G. Deleuze, *Difference and Repetition*, tr. P. Patton (London: Continuum, 1997). Along with Ferdinand Gonseth and very recently Jean Largeault, Gilles Deleuze is one of the (too) rare philosophers to have appreciated the importance of Lautman.

32. Lautman, *Essai*, 211.

33. Ibid., 142 [emphasis ours].

34. Ibid., 142–3 [emphasis ours].

35. Ibid., 142 [emphasis ours].

36. Ibid.,226.

37. Ibid.,206.

38. Ibid.,226.

existence of a domain from within which the objects of scientific knowledge draw their life and their matter.[39]

Thus transcendentally understood, the transformation of comprehension into genesis permits the articulation between the *transcendence* of Ideas and the *immanence* of the schemas of the associated structures.

> There exists [...] an intimate link between the transcendence of Ideas and the immanence of the logical structure of the solution of a dialectical problem within mathematics; it is the notion of genesis that will give us this link.[40]

More precisely, here genesis means a relation to foundation and to origin (as in every dialectics):

> The order implied by the notion of genesis is not [...] the order of the logical reconstruction of mathematics, in the sense in which all the propositions of a theory unfold from its initial axioms—for dialectics is not a part of mathematics, and its notions are not related to the primitive notions of a theory. [...] The anteriority of the Dialectic [is] that of 'concern' [souci] or of the 'question' in relation to the response. It is a question here of an 'ontological' anteriority, to take up an expression of Heidegger's, exactly comparable with that of 'intention' in relation to a plan.[41]

The philosopher has neither to find laws, nor to predict a future evolution; his role consists uniquely in becoming conscious of the logical drama at play within theories

We might ask whether, for Lautman, there is not an intersection here between a historical dialectic and a phenomenology of correlation. It is as if, in their 'urgency', the problems formulated by Ideas admit as their *intentional correlate* the theories in which they are concretised and historicised. The Ideas reflect a consciousness, a becoming-conscious-of:

> The philosopher has neither to find out the laws, nor to predict a future evolution; his role consists uniquely in becoming conscious of the logical drama at play within theories. The only a priori element that we will conceive of is given in the experience of this urgency of problems, anterior to the discovery of their solutions.[42]

It is this intentional content of Ideas that renders them at once transcendent and immanent to the mathematical field.

> Qua problems posed, relative to liaisons that are susceptible of supporting between them certain dialectical notions, the Ideas of this Dialectic are certainly transcendent (in the usual sense) in relation to mathematics. On the contrary, since every effort to give a response to the problem of these liaisons, by the very nature of things, yields the constitution of effective mathematical theories, we are justified in interpreting the overall structure of these theories in terms of immanence for the logical schema of the solution sought.[43]

(d) Metamathematics, Platonism, Ontological Difference, Imitation and Expression

As correlation between the 'proper movement' of mathematical theories and 'the liaisons of ideas which are incarnated in that movement', as genetic reality defined in transcendental fashion 'as the advent of notions relating to the concrete within the analysis of the idea',[44] the 'inherent reality' in mathematics[45] is thought by Lautman on the basis of major philosophical traditions which he brings together in a wholly original fashion:

i. The Platonic tradition of the participation of the 'sensible' (here, mathematical idealities) in the 'intelligible' (here, Ideas). Lautman traces this all the way into Leibnizian metaphysics.

ii. The Kantian tradition of constitution. Here the situation is quite complex, in so far as the relation between 'sensible' and 'intelligible' becomes

39. Ibid.,206.
40. Ibid., 212.
41. Ibid., 210.
42. Ibid., 142.
43. Ibid., 212.
44. Ibid., 205.
45. Ibid., 205.

that between transcendental aesthetic and transcendental analytic, with mathematics playing a constitutive rather than a dialectical role. Now, for Lautman, as we have seen, through the history of mathematics, a *Dialectic of the Concept becomes transcendentally constitutive*. With such a theoretical gesture come great difficulties in evaluation. For, although Platonist, Lautman's dialectic is obviously not unrelated to transcendental dialectic (just consider the link between the thematic opposition continuous/discrete and the second antinomy). To render it transcendentally constitutive is thus in some way to *historicise the a priori* and, more precisely, in so far as mathematics exercises a schematising function relative to the categories of diverse regional ontologies, to historicise the schematism.

Whereas Hegel affirms contradiction in the concept alone, Lautman affirms the labour of the speculative within the physico-mathematical itself

iii. Whence Lautman's highly ambiguous relation to Hegel. In Lautman we rediscover the speculative Hegelian conception of contradiction as the life of the concept and the movement of reason. But whereas Hegel affirms contradiction in the concept alone, independently of all relation to Kantian formal objectivity, and thus independently of any mathematics or physics, Lautman on the contrary affirms the labour of the speculative within the physico-mathematical itself.

iv. Finally, as we have seen, there is also a strictly phenomenological component in Lautman's conception of the mathematical real. Like Cavaillès, Lautman comes back to a critico-phenomenological conception of objectivity—that is to say, to the question of transcendental logic. But he critiques phenomenology in its guise as a philosophy of consciousness reflexively regressing towards a constitutive *subjectivity*.

In order to clarify these various points, let us further develop three particularly delicate motifs.

The passage from metamathematics to metaphysics. The reference to Hilbert's axiomatic structuralism is foundational for Lautman. But through an authentically speculative gesture, Lautman will considerably enlarge the field of the significance of metamathematics.

Metamathematics examines mathematical theories from the point of view of concepts such as those of non-contradiction or completeness, which are not *defined*—and 'this is very important'[46] —within the formalisms to which they are applied. Now, such concepts are more numerous than might appear. There exist

> other logical notions, equally susceptible of being potentially linked to each other within a mathematical theory, and which are such that, contrary to the preceding cases [of non-contradiction and completeness], the mathematical solutions of the problems they pose can comprise an infinity of degrees.[47]

Thus the dialectical Ideas rethink metamathematics in metaphysical terms and, in doing so, extend metaphysical governance to mathematics.

The question of Platonism. In the conclusion of his major thesis, when he discusses Boutroux's work *The Scientific Ideal of Mathematicians*, Lautman broaches the question of Platonism—that is to say, of the reality of mathematical idealities. For Boutroux, as for Brunschvicg and for the great majority of mathematicians, there exists an objective mathematical real. Although this real is not that of 'external perception' or 'inner sense',[48] this doesn't mean that mathematics is a meaningless symbolic language, as logicist nominalism would have us believe. There are mathematical facts (the irrationality of $\sqrt{2}$, the transcendence of e and of π, the fact that the (Abelian) integral $\int dx/\sqrt{P(x)}$ is not elementarily integrable if $P(x)$ is a polynomial of degree ≥ 3, the truth or falsity of the Riemann hypothesis)—facts which appear to be 'independent of the scientific construction'[49] and as if endowed with an objective transcendence analogous to that of physical facts. This is why, according to Boutroux, 'we are forced to attribute a true objectivity to mathematical *notions*'.[50] The aporia of Platonism thus stems from

46. Ibid., 206.
47. Ibid., 28 [emphasis ours].
48. Ibid., 24.
49. Ibid., 136.
50. Ibid.

the conflict between realist and nominalism in the conception of objectivity:

1. If we conceive objectivity as a purely transcendent exteriority, we will, like Boutroux, adopt a realist position giving us intuitable mathematical facts that are independent of any language in which we might formulate them: 'the mathematical fact is independent of any logical or algebraic clothing in which we might seek to represent it'.[51]

2. If, on the other hand, we conceive this objectivity as pure construction, we will, like the logicists, adopt a nominalist position according to which the mathematical real is purely a being of language.

But the mathematical real is obviously too subtle to be thought through such a naive antinomy.

1. Firstly, the objectivity of mathematical idealities (which is not in doubt) cannot be separated from the formal languages in which they are expresed, for there is 'an essential dependence of the properties of a mathematical being upon the axiomatic of the domain to which it belongs'.[52]

2. And then, as we have seen above, mathematical facts are organised into concepts, and then into theories, and 'the movement of these theories incarnates the schema of liaisons of certain Ideas'.[53] By virtue of which the mathematical real depends not only upon the factual basis of mathematical facts, but equally 'upon the global intuition of a suprasensible being'.[54]

To which we must add a more technical aspect of Platonism, concerning the possibility of mastering mathematical entities in a manner at once ontological and finitary:

> In the debate opened up between formalists and intuitionists, since the discovery of the transfinite, mathematicians have tended to designate summarily under the name of Platonism every philosophy for which the existence of a mathematical being is assumed, even if this being cannot be constructed in a finite number of steps.[55]

But despite the delicate constructivist problems with which it is associated, we remain here within a 'superficial conception of Platonism'.[56]

As far as we are concerned, the most adequate response to the aporia of Platonism seems to lie in the Husserlian principle of *noetic-noematic correlation*, which allows that the transcendence of objects is founded in the immanence of acts. According to this principle, the rules of the noetic syntheses of acts (regardless of the syntactical rules providing the norm for symbolic usage, in the theory of formal languages or of eidetico-constitutive rules as in transcendental phenomenology) can admit as noematic correlates objective idealities which 'resist', and which manifest all the urdoxic characteristics of reality manifested by transcendent objects. If one does not take up a thinking of correlation, one must either make of the noema real (non-intentional) components of acts, thus ending up with a subjectivist idealism; or hypostasise them into subsistent transcendent objects, thus ending up with an objectivist realism.

In *Mathematical Idealities* Jean Toussaint Desanti has shown very well, with several examples (the construction of the continuum and the Cantorian theory of sets of points), how to develop an analysis of mathematical objects as intentional objects. Following Husserl, Cavaillès, and Bachelard, he has shown how through abstraction one extracts out of the field of objects common 'normative schemas' and 'operatory kernels' which correspond to so many axiomatisable structural concepts; and how through thematisation one transforms properties into new objects. The objects constructed in this way are not intuitable as such. They do not have a 'transparent essence'. They are rationally authorised objects, axiomatically governed but not given intuitively (the critique of Husserlian given intuitions).[57] Desanti insists on this crucial point, distinguishing as different types of acts the 'positing of explicit kernels' and 'horizonal positing'. In the act of positing

51. Ibid.,139.

52. Ibid.

53. Ibid., 135.

54. Ibid., 136.

55. Ibid., 143.

56. Cf. J.T. Desanti, *Les Idéalités mathématiques* (Paris: Seuil, 1968), 48–9.

57. Ibid., 97.

explicit kernels, there is a 'grasp of the kernel in a consciousness of apodictic and direct self-evidence, yielding the reflexive immanent character of its own self-evidence.' There is indeed intuition, but it is a modality of action that admits as objectal correlate an 'explicit kernel', a noematic object, not a subsistent object given intuitively 'in person'. The object here is an intentional object, which can only partially be fulfilled in intuition; an object whose 'transparency' is produced 'in a modality of the act of positing'. So here, self-evidence is not 'a mode of specific apprehension', but a positing—that is to say the product of a process of bringing to light. When an act of positing (definitions, axioms, etc.) 'delimits the posited once and for all', 'the consciousness of self-evidence which lies reflexively at the heart of the act is here only a phenomenologically immanent character specific to the mode in which, at that moment, the constitution of the object, of consciousness within its object, is installed.'[58] Thus, through reflection on the immanence of acts, mathematical idealities appear as intentional objects, that is to say as noematic poles, poles of ideal unity, poles normative for rule-governed sequences of acts. The reality of their existence 'is constituted in the unity of three moments':[59] t he moment of the hypothetical object associated with operations and procedures of a certain type, the moment of the object as noematic pole of unity, and the moment of the rule-governed and axiomatised mathematical object. It is the second moment that is essential, in so far as it operates the passage from the first to the third. Now, qua intentional, this moment is extralogical or extramathematical.

We can thus say that, in structural mathematics, the axiomatic formalises intentionality. As Desanti affirms magnificently, intentionality is 'the mode of being of the consciousness of the object at the heart of its objects'. The intentional kernel of the object is a movement of 'double mediation' linked to the 'bipolarity' of the object in the a priori of correlation. It is 'neither pure positing of normative ideality', 'nor simple consciousness of being assigned to a non-governable becoming'. 'It is positing of the pure possibility of sequences of acts capable of effectuating, within a field of intuition no longer governed, the verifications demanded by the positing of normative ideality.' The expression 'intentional core' designates here 'that moment where the consciousness of the object grasps an object as the essential unity of a norm and of an unfinishedness', 'the synthetic moment in which the object manifests the circular relation of its ideality and its becoming', 'the inseparable unity of a norm and a becoming.'[60]

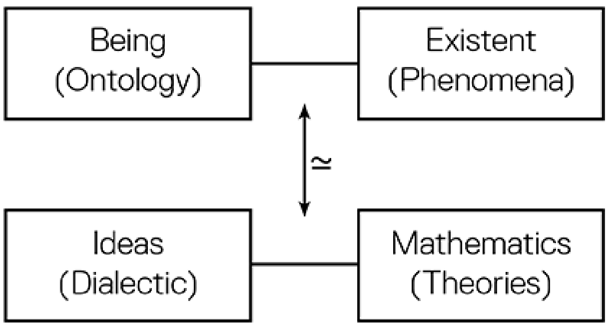
On this basis, Desanti developed an intentional analysis not only of objects but also of theories and of the 'consciousness of the axiom'. The latter is essential for clarifying the profound solidarity between Husserlian phenomenology and Hilbertian axiomatics, and in particular allows us to clarify—and, we believe, even to resolve—the aporia of Platonism.

Mathematical truth participates in the temporal character of the mind, for 'Ideas are not the immobile and irreducible essences of an intelligible world'. Their dialectic is historical

Mathematical Idealities might be considered an indispensable complement to Lautman's oeuvre in so far as it is precisely on the question of the 'proper movement' of theories that, in Lautman, the intentional phenomenological analysis comes together with the Platonist dialectic in a 'phenomenological description of concern for a mode of liaison between two ideas'.[61] In their twofold status as intentional correlates and horizons of becoming, mathematical theories do not develop linearly 'as an indefinitely progressive and unifying extension'.[62] They 'are rather more like organic units, lending themselves to those global metamathematical considerations which Hilbert's oeuvre announces'.[63] Through the associated Ideas, 'mathematical truth [...] participates in the temporal character of the mind',[64] for 'Ideas are not the immobile and irreducible essences of an intelligible world'.[65] Their dialectic is, let us emphasise once more, *historical*.

58. Ibid, 84.
59. Ibid, 92–3.
60. Ibid., 92–3.
61. Lautman, 142.
62. Ibid., 140.
63. Ibid.
64. Ibid.
65. Ibid., 143

Ontological difference. In regard to the relation between comprehension and genesis (see §2. C) which results from the 'governance' of mathematics by a superior dialectic, Lautman situates himself explicitly within a transcendental perspective: 'it is through a "transcendental" interpretation of the relation of governance that one can better take account of this involvement of the abstract in the genesis of the concrete'.[66] In what follows, let us insist upon the parallel established by Lautman:



Dialectical Ideas are to mathematical theories what being and the meaning of being are to beings and to the existence of the being (ontological difference). The fact that 'the adequate comprehension' of Ideas and their 'internal liaisons' should 'give rise to systems of more concrete notions wherein those liaisons are affirmed' responds to the Heidegerrian affirmation that 'the production of notions relative to concrete existence is born of an effort to comprehend more abstract concepts'. 'The advent of notions relative to the concrete within an analysis of the Idea' responds to the fact that the truth of being is ontological, and that the existent that manifests itself can only reveal itself in conformity with the comprehension of the structure of its being. In this Heideggerian reinterpretation of Platonism and transcendental logic, we arrive back at historicity, in so far as, for Heidegger, being is identified with the historiality of its meaning: 'conceptual analysis necessarily ends up projecting, as if ahead of the concept, the concrete notions in which it is realised or historialised.'[67]

Much could be said here on Lautman's usage of Heidegger against the backdrop of a remarkable absence of reference to Hegel.

i. Certainly, just as Heidegger conceived metaphysical systems as so many responses to the question of the meaning of being, responses each time oriented towards beings and not towards the comprehension of being, which remained unthought in them (the play of the veiling-unveiling of *aletheia*), so Lautman conceived mathematical theories as so many responses to Ideas, responses always oriented towards mathematical facts and objects and not towards the comprehension of Ideas themselves, which remained unthought in these theories. And yet, as Barbara Cassin points out, in Heidegger ontological difference cannot be seen as homologous with the opposition between Essence and Existence. For the latter (like the opposition between transcendence and immanence) is *metaphysical*. The Heideggerian ontological difference between being and beings cannot be made homologous with any metaphysical difference. One cannot therefore use any such metaphysical difference to speak either of Heideggerian difference or of the relations between it and the history of the systems of responses that it has engendered.

ii. There is a problem of 'metalanguage' here which, as we know, led Heidegger (not to mention Derrida) to break with the metaphysical style. There is no metalanguage capable of speaking adequately of ontological difference.

iii. But we must remark that this problem is not pertinent for Lautman. For in so far as he treats of mathematical theories and not metaphysical systems, for him metaphysical languages can, and indeed (as we have seen) do constitute an adequate metalanguage.

iv. Finally, the reference to Heidegger again accentuates the ambiguity of Lautman's relation to Plato, Hegel, and Husserl, in so far as Heidegger himself maintains an ambiguous relationship to these decisive moments of thought. In particular, here we should deepen the analogies between the Hegelian dialectic and Heideggerian historiality.

(e) The Lautman/Cavaillès Debate of 4 Feb 1939

These few elements of Lautman's philosophy take on a singular aspect when one observes them at work in the debate—one of rare intensity—that brought

66. Ibid., 205.

67. Ibid., 206.

together Lautman and Cavaillès at the Societé Française de Philosophie. Present, amongst others, were Henri Cartan, Paul Levy, Maurice Fréchet, Charles Ehresmann, and Jean Hyppolite. It was the February 4, 1939. Six years to the day before Yalta....

Cavaillès begins by recalling how Hilbertian metamathematics had internalised the epistemological problem of foundation by transforming it into a purely mathematical problem. He thus upholds four theses.[68]

i. Mathematics has a solidarity—a *unity*—that prevents any regression to a supposedly absolute beginning (this being a critique both of logicism and of a phenomenology of the origin developed within the framework of a philosophy of consciousness).

ii. Mathematics develops according to a singular, autonomous, and originarily unforeseeable becoming—thus, an authentically dialectical becoming.

iii. The resolution of a problem is analogous to an *experiment* that is effective, as a programme, through the sanction of rule-governed acts. Mathematical activity is an experimental activity—in other words, a system of acts legislated by rules and subject to conditions that are independent from them.

iv. In mathematics, the existence of objects is correlative with the actualisation of a method. It is non-categorical,[69] and proceeds from the very reality of the act of knowing. As correlates of acts, the objects project into representation the steps of a dialectical development. Their self-evidence is conditioned by the method itself.

To these theses, which he largely shares, Lautman responds by placing the accent on the question of *Sense*. The manifestation of an existent in act 'only takes on its full sense' as a response to a preceding problem concerning the possibility of this existent; this is why the establishing of effective mathematical relations appears to be rationally posterior to the problem of the possibility of such liaisons in general. Lautman then sums up the way in which the ideal Dialectic, 'presenting the spectacle' of the genesis of the Real out of the Idea, organises the concrete history of mathematics under 'the unity of themes'. It is of course on the question of Sense—in other words, of the participation with the intelligible—that his disagreement with Cavaillès comes to light. For Cavaillès, there are no general characteristics constitutive of mathematical reality.[70] For Lautman, on the contrary,

> [the] objectivity of mathematical beings [...] only reveals its true sense within a theory of the participation of mathematics in a higher and more hidden reality [which] constitutes the true world of ideas.[71]

Mathematics are a 'mixture' wherein a passage from essence to existence takes place, dialectically. Lautman repeats:

> One passes insensibly from the comnprehension of a dialectical problem to the genesis of a universe of mathematical notions, and it is the recognition of this moment when the idea gives birth to the real that, in my view, mathematical philosophy must aim at.[72]

Here Dialectics is converted naturally into a research programme, an ambitious programme which Lautman formulates with remarkable simplicity and sobriety by inscribing it into the Platonist, critical, and phenomenological traditions of idealism:

> We thus see what the task of mathematical philosophy, and even of the philosophy of science in general, must be. A theory of Ideas is to be constructed, and this necessitates three types of research: that which belongs to what Husserl calls descriptive eidetics—that is to say the description of these ideal structures, incarnated in mathematics, whose riches are inexhaustible. The spectacle of each of these structures is, in every case, more than just a new example added to support the same thesis, for there is no

68. One will recognise those indicated above under the heading 'The Question of Platonism'.

69. Here Cavaillès draws the philosophical consequences of the results of non-categoricity in the logical theory of models (Skolem's paradox, syntactic/semantic divergence, existence of non-standard models). For an elementary introduction to these questions, consult J. Petitot, 'Infinitesimale', in *Enciclopedia Einaudi*, VII (Turin: Einaudi, 1979), 443–521.

70. We saw how, in *On the Logic and Theory of Science*, Cavaillès opened up this transcendental problematic.

71. <http://www.urbanomic.com/document/mathematical-thought/>.

72. Ibid.

saying that it might not be possible—and here is the second of the tasks we assign to mathematical philosophy—to establish a hierarchy of ideas, and a theory of the genesis of ideas from out of each other, as Plato envisaged. It remains, finally, and this is the third of the tasks I spoke of, to remake the Timaeus—that is, to show, within ideas themselves, the reasons for their applicability to the sensible universe.[73]

To remake the Timaeus—that is, to show, within ideas themselves, the reasons for their applicability to the sensible universe

At the time of the debate, opinion on Lautman was largely unfavourable: the mathematicians avowed their confusion as to 'philosophical speculation' and its incomprehensible 'subtleties', and the philosophers reproached him for a certain imprecision in his use of the term 'dialectic'.[74] There was a clear consensus that philosophy, when confronted by mathematics, must either submit or be dislocated. Hyppolite, obliged to represent philosophy, even goes so far as to affirm: 'As to M. Lautman's thesis, one may well fear, in adopting it, that mathematical notions would evaporate, in a certain way, into pure theoretical problems that surpass them.'

In their responses (in particular to Fréchet, who had maintained 'naive' realist theses), Cavaillès and Lautman both situated themselves in a transcendental perspective. Firstly Cavaillès:

> I do not seek to define mathematics, but, by way of mathematics, to know what it means to know, to think; this is basically, very modestly reprised, the question that Kant posed. Mathematical knowledge is central for understanding what knowledge is.

Then Lautman:

> The genesis of which I spoke is thus transcendental and not empirical, to take up Kant's vocabulary.

To say (as Fréchet affirmed) that it is the (physical) Real that engenders the (mathematical) Idea and not the inverse, is to think the Idea *via abstraction* and thus to confuse it with an empirical concept. Now, as 'conception of problems of structure', Ideas are the autonomous transcendental concepts 'in relation to the contingent elaboration of particular mathematical solutions'.[75]

Ultimately, in his final response to Cavaillès, Lautman comes back once more to the question of Sense, to the 'admirable spectacle' of an ideal reality transcending mathematics and, above all, independent of the activity of the mind (opposition between a Dialectic of the concept and a philosophy of consciousness).

> The precise point of our disagreement bears not on the nature of mathematical experience, but on its meaning and its import. That this experience should be the condition sine qua non of mathematical thought, this is certain; but I think we must find in experience something else and something more than experience; we must grasp, beyond the temporal circumstances of discovery, the ideal reality that alone is capable of giving its sense and its status to mathematical experience.

Beyond its moving spiritual significance, this historical debate shows that the knot of the Dialectic consists in making the problematic of the constitution of objective realities equivalent to a hermeneutics of the autonomous historical becoming of mathematics. This point can only be clarified through an evaluation of Lautman's philosophy, by which we might hope to mitigate his tragic premature death.

73. Ibid.

74. Even though, as we have already seen Barbara Cassin remark, the 'dialectic' developed by Plato in Republic concerned the contemplation of Ideas and, by virtue of this, did not have the controversial and antinomic character it took on in the metaphysical tradition from Aristotle to Kant. As much as Lautman is authentically Platonist in his conception of the participation of the sensible in the intelligible, he also seems to become surreptitiously Kantian and/or Hegelian in his conception of Dialectic as Antithetic.

75. As for us, the criticism we would make of Lautman is that of not having clearly divided transcendental concepts into determinant categories and rational Ideas. In the justification: 'M. Hyppolite says that posing a problem is not conceiving anything; I respond, after Heidegger, that it is to already delimit the field of the existent' (*Mathematical Thought*), we find a categorial Analytic ('delimit the field of the existent') amalgamated with a rational Antithetic.

3. Metamathematical Dialectic and Thematic Analysis

How are we to evaluate the Lautmannian conception, both on the plane of mathematical philosophy and that of transcendental philosophy (the relation between metaphysics, reality, and mathematics in the framework of a constitutive doctrine of objectivities)?

One of the first remarks one might make would be to note that Lautman developed a comprehensive—hermeneutic—analysis of mathematics that might be qualified as thematic in Gérard Holton's sense.

On the basis of the historical study of numerous concrete cases, Holton discovered empirically and inductively the existence of certain dialectical premises and presuppositions underlying scientific representations and practices, and acting unconsciously in the genesis of scientists' work. He called these generally occulted formations of sense *themata*, and developed a psycho-historical and sociocultural version of transcendental dialectic. As a system of conflicts between opposing notions—as problematic Ideas—the *themata* develop an antithetic of objective reason. They are non-refutable, and manifest a certain stability even if, obviously, the evolution of the sciences leads to considerable variations in their determination.

According to Holton, the thematic analysis of the rational conflicts (antinomies, even) between discrete/continuous, simplicity/complexity, analysis/synthesis, mechanism/finalism, determinism/indeterminism, holism/reductionism, constancy/evolution/sudden transition, etc. pertain to an investigation of the scientific imagination and, through their dialectical nature, could help account for the conflicts between different schools of thought. His orientation is thus psychological (imagination), sociological (controversy), and historical (empirical case studies)—in short, anthropo-semiotic rather than epistemological and gnoseological. As Angèle Kremer Mariette has noted,[76] his point of view is that of an 'anthropology of science resting on an essentially genetic epistemology'. He envisages the activity of the scientist as 'a development of symbolisation, on the basis of a real apprehended according to certain forms acceptable to a given state of a society and a history'. In this sense, he is rather close to a hermeneutico-communicational (Habermasian) analysis of beliefs repressed through the formation of consensus.

One might then say that, in a context where positivist dogmatism had reigned triumphant, Lautman provided the bases for a *thematic analysis of pure mathematics*. This is in itself already of great importance:

i. For the history of ideas;

ii. For the study of the mathematical imagination in its relation to diverse sociocultural symbolic formations (a surpassing of the traditional opposition between respectively internalist and externalist points of view);

iii. For the isolation of mathematical creativity's solidarities with the movement of thought.

Lautman's project is not anthropological or historicist, but metaphysical and rationalist

But this remains largely insufficient. Lautman's project, unlike Holton's, is not anthropological or historicist, but metaphysical and rationalist. It bears not upon the activity of the epistemic subject, but upon the reality of the theoretical object. As we have seen, it has an *ontological* import, and must be evaluated in transcendental terms. But to speak in terms of a transcendental logic (a logic of the objectivity of the object of knowledge) of a dialectic of the concept immanent to the development of objective theories, is to admit an *aporetic* ground of the real. It is to admit what René Thom, precisely speaking of Holton's thematic analysis, called the 'foundational aporia' constitutive of the real.

In his article 'Holton's Themes and Foundational Aporias',[77] René Thom indicated how the *themata* can be reconstructed by combining the dyads unity/diversity and extension/quality with the action of time in the empirical manifold. It is principally the irreducible tension between the antagonistic metaphysical principles of unification and diversification that are found at the origin of irreducible aporias (such as the discrete and the continuous, space and

76. Article 'Holton' in the *Dictionnaire des Philosophes*.

77. R. Thom, 'Holton's Themes and Foundational Aporias', in *Logos et Théorie des catastrophes* (Colloque de Cerisy, 1982).

matter, etc.), to which specific and effective theories can be considered as so many partial solutions, always local and always provisional. We do indeed find here, implicitly, the Lautmannian concept of problematic dialectical Ideas.

But the principal difficulty remains: that of the intersection *between objectivity and sense*, that is to say between a transcendental thinking of the constitution of objects and a hermeneutic thinking of the historical becoming of theories. It is this last point we now wish to investigate.

4. Mathematics and Reality: Transcendental Schematism as Symbolisation[78]

(a) The Central Question

We consider that in fact Albert Lautman described one of the rare philosophical conceptions of the relation *between mathematics and reality*—perhaps the only one—to be compatible with the two following characteristics of our modernity:

i. As far as mathematics is concerned: in particular the *autonomisation* and *unification* of mathematics, that is to say not only its being torn from the sensible world of prepredicative givens (from which mathematics had long been considered to derive via idealisations and successive abstractions), but equally its emancipation from empirical experience (cf. 2. b).

ii. As far as reality is concerned: the possibility of generalising to diverse regional ontologies, in conformity with the constitutive programme of phenomenology, the critical doctrine of the constitution of objectivities.

The central question is, let us recall, the following (cf. 2. c and 2. d): How, in its being and its autonomous becoming, can mathematics continue to be implicated in a determining fashion within the fields of transcendentally constituted objectivity? In the transcendental conception of objectivity, metaphysics has been articulated with physics through three intermediary instances:

i. An aesthetic permitting the separation of the empirical real from reality in itself, and thus the distinction between an analytic and a dialectic;

ii. A schematism permitting the conversion of an analytic of concepts into an analytic of principles;

iii. A mathematics affined to the aesthetic, and which comes to determine its forms of intuition into formal intuitions.

How can such a conception be reprised, developed, diversified, rectified even, if, on one hand, mathematics are autonomised and if, on the other, in order to be able to generalise the Kantian gesture, one is constrained, like Husserl, to subordinate diverse regional ontologies to a formal ontology—that is to say, to subordinate the aesthetic to an analytic, one that is purely logical and thus, as Cavaillès insisted, 'irremediably insufficient'? The difficulty is such that, in fact, it would be easier to eliminate the question rather than seek a response to it. On this point logical empiricism and various post-positivist scepticisms were in complete agreement.

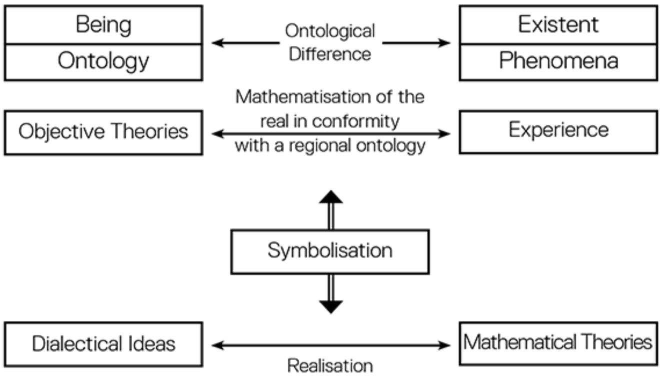
And yet we find the first element of a response in Lautman, in one of the concluding assertions of his major thesis: '*The process of the liaison of theory and experience symbolises the liaison of Ideas and mathematical theories.*'[79]

(b) Symbolisation and Constitution: Towards a Hermeneutics of Objectivity

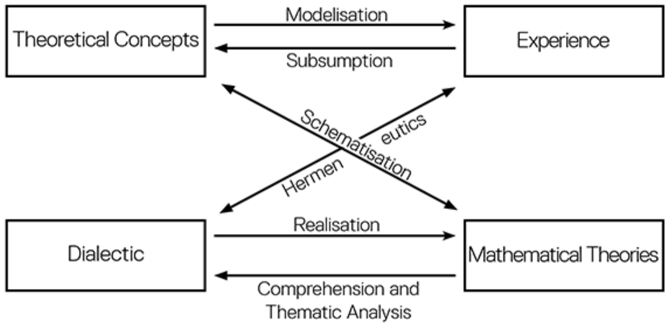
In our view, Lautman's aphorism represents one of the most fulgurating thoughts in the philosophy of the modern sciences. It sets up a parallel, a proportion, an analogy, in regard to ontological difference. What we have already said in 2. c and 2. d here takes on its full significance.

78. Constraints of space prevent us from developing in detail these delicate technical points. The interested reader is referred to J. Petitot, 'À propos de « Logos et Théorie des Catastrophes »', *Babylone* 2/3 (Paris: Christian Bourgois, 1983), 221–260; *Morphogenèse du Sens II* (Paris: PUF, 1986); 'Apories fondatrices et Dialectique mathématique', Conference 'Controverses scientifiques et philosophiques', University of Evora. Documents du Centre d'Analyse et de Mathématiques sociales (Paris: École des Hautes Études en Sciences sociales, 1986); 'Mathématique et Ontologie', in *La rinascita della filosofia della scienza e della storia della scienza in Italia dagli anni trenta ad oggi*, University of Varese; and 'Schématisme et Interprétation', in *Colloque sur l'Interprétation* (Collège Internationale de Philosophie, 1986).

79. Lautman, 146 [emphasis ours].



Let us remark, then, that for Lautman, as for every rationalist, no scientific concept is authentically theoretical unless it is endowed with a mathematical meaning. In science, to use theoretical concepts to speak of certain types of objects, properties, or situations, is to have chosen a universe of mathematical discourse: to speak of simultaneous phenomena, for example, is to speak the language of special relativity; to speak of co-measurable magnitudes is to speak the language of commutative operators; to speak of the invariance of a magnitude is to speak the language of group theory, etc. Consequently, in the transcendental analogy proposed by Lautman, mathematics intervene in the position of a *middle term*, linking the *Dialectic* to *phenomenal experience*. To be more precise, the relation between mathematical theories and objective theories is effectuated through the conversion of the semanticism of fundamental concepts into explicit mathematical constructions. We have shown elsewhere why and how such a conversion can be interpreted as a *schematisation* (in the Kantian sense: the construction of a concept into a determinate pure mathematical intuition). The schematisation of concepts is the key to authentic theorisations. All conceptual science must, at a certain moment, be able to redeploy in a *constructed diversity of models* the movement of subsumption of the empirical (given) manifold under the unity of concepts. For this to take place, the semanticism of fundamental concepts must be able to become a source of models. There must be a *generativity*, and thus a mathematisation. It is this that schematisation gives us. For with schematisation the models of the phenomena of a certain region come 'into conformity with the things themselves', that is to say into conformity with a categorially-determined objective essence.

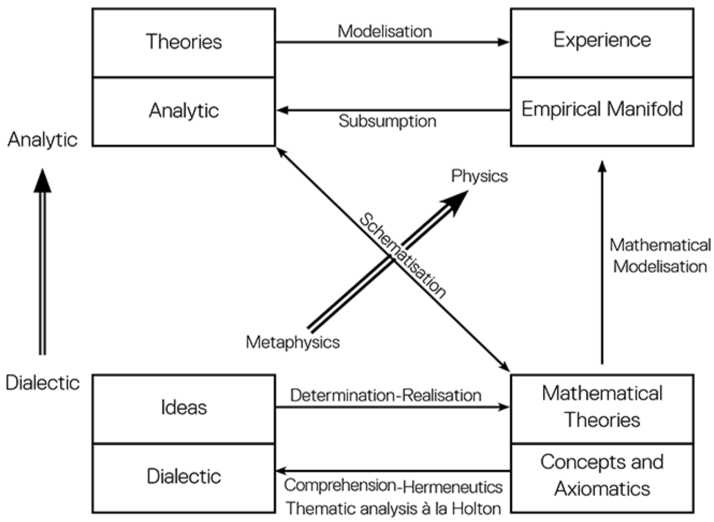


Through the transcendental analogy that is 'symbolisation', the Dialectic is thus converted into a *hermeneutic*, not only of mathematics, but, more profoundly, of *objectivity*. Thus Lautman resolves the central problem of the unity of sense and being in a transcendental doctrine where being is identified with the constitution of objectivity. Through their twofold function, that of:

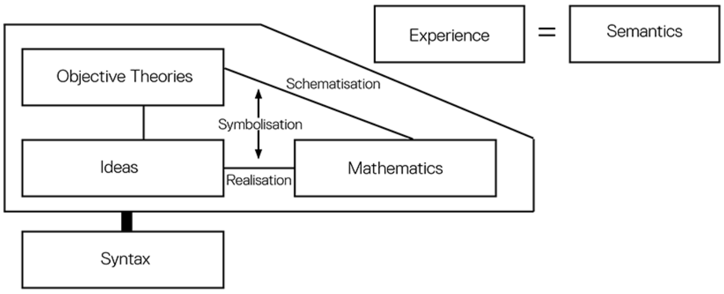
i. Transforming the semantic content of theoretical concepts into the source of models for phenomena (schematisation);

ii. Realising a Dialectic of the concept

—with (i) and (ii) being linked by a symbolisation, mathematics engender—in their autonomous theoretical becoming—*aesthetics* (plural) and *schematisms* (plural) for an open and indefinite number of regional ontologies. They progressively transform an ideal dialectic into a concrete and plural history of transcendental analytics. In their relation to reality, they historicise the Kantian operation into a generalised Critique which represents a dynamic version of constitutive phenomenology.



In reducing mathematics to being nothing more than the syntax of languages in which verifiable experimental statements can be made, in identifying comprehension and intelligibility with a 'mystical belief', in liquidating the synthetic a priori, logical empiricism and neo-positivism make this schema fall back onto a simple dualism of syntax/semantics similar to that encountered within the logical theory of models.



(c) An Example

To make the transcendental analogy clearer, let's take up some elements of Lautman's superb text on 'Symmetry and Dissymmetry in Mathematics and Physics'.

Having recalled the Kantian (Anti-Leibnizian) proposition which led to the Transcendental Aesthetic (namely, that the incongruence of symmetrical figures is intuitive, not conceptual), and having recalled the importance of enantiomorphies in crystallography, biology (Pasteur), and the physics of matter (Curie), Lautman proposes to show in detail how it is indeed the mathematical deployment of fundamental concepts such as symmetry that governs theoretical physics. It thus exemplifies the way in which the defined mathematical real metaphysically marries with physical reality in order to implicate it normatively and constitutively. With the determinant role given to the concept, we are truly very far from the empiricist and positivist theses according to which mathematics is just a formal construction without ontological import, and is reduced (as in Carnap) to being nothing more than the logical syntax of reductionist explanation. The fundamental reference, once more, is to the *Timaeus*:

> The materials from which the Universe is formed are not so much the atoms and molecules of physical theory as the great couples of ideal contraries such as the Same and the Other, the Symmetrical and the Dissymmetrical, associated with each other through the laws of a harmonious mixture.[80]

In analysing precise examples such as the action of space-time symmetries on Dirac's spinors in special relativity, the symmetry/asymmetry of wave functions in quantum mechanics (Bose-Einstein and Fermi-Dirac statistics), and Birkhoff and von Neumann's use of abstract duality theory in quantum logic, Lautman insists repeatedly on the fact that 'the distinction between right and left in the sensible world can symbolise the non-commutativity of certain abstract operations of algebra',[81] and that such a symbolisation is, on the plane of principles and of conditions of possibility, far more determinant than a precise relation between the description of objective facts and certain ad hoc mathematical structures. If a mathematical physics is possible—if, for example, a differential geometry can become (as in general relativity) a cosmology or if a theory of operators can become (as in quantum mechanics) a theory of physical observables—it is because on the plane of pure concepts there is an 'analogy' between mathematics and physics, their agreement being 'the proof of the intelligibility of the universe'[82] and of the 'penetration of the real by intelligence'.[83]

If a mathematical physics is possible it is because on the plane of pure concepts there is an 'analogy' between mathematics and physics, their agreement being 'the proof of the intelligibility of the universe'

From the moment one posits a participation of theories in a transcendent intelligible world, the common participation of two theories in the same Idea manifests ipso facto, through the unity of the latter, an essential solidarity and accord between the former, whether it is a question of two mathematical theories or of a mathematical theory and a physical theory.

80. Ibid., 241.
81. Ibid., 244.
82. Ibid., 284.
83. Ibid.

> A common participation in one and the same dialectical structure thus brings to light an analogy between the structure of the sensible world and those of mathematics, and allows us better to comprehend how these two realities are in accordance one with one another.[84]

Whence the fundamental affirmation that the theme symmetry/dissymmetry exemplifies 'the sensible manifestation of a dialectical structure which is generative of abstract mathematical realities just as it is of conditions of existence for the universe of phenomena'[85]—this link between spatiality and Ideas being 'perhaps the most current sense that the notion of intelligible extension can take on in our time'.[86]

We share these theses up to a point, except that, for us, analogy is a schematisation rather than a symbolisation, a point which involves the whole Lautmannian conception of the transcendental and thus of objectivity. As we have indicated above (4.b), the schematism of the categories of a regional ontology remains the key to the comprehension of the relations between mathematics and reality. Cavaillès understood this admirably, as did Gonseth after him.[87] But in a modern doctrine of objectivity, we must extend and rectify Kant on many points.[88]

i. First of all, we must *invert* the relation of dependence between metaphysical exposition and transcendental exposition in the Transcendental Aesthetic, making of formal intuitions the *evolutive* mathematical determinations of forms of intuition.

ii. Consequently, as far as schematism is concerned, we must make it depend not only, as in Kant, upon the metaphysical exposition, but equally upon the transcendental exposition, thus making it the origin of the mathematical organon in the sciences (cf., in 2.a, our remarks on the Kantian interpretation of Poincaré's conventionalism).

iii. In so doing, the schematism, contrary to what it is in Kant, becomes the *construction* of the concept.

iv. But this obviously does not mean that the ontological categories are as such constructible mathematical concepts. It means that they can be analogically homologous to such concepts.

v. This constructive schematism extends the categories and metaphysical principles associated with derived concepts operating upon empirical contents.

The concept of symmetry furnishes a prototypical example. Playing a central role in the Transcendental Aesthetic, it is a categorial concept of nature which is found schematised in various ways. In each case, its construction rests upon its *interpretation* within a certain mathematical universe. Now, such an interpretation is not a symbolisation. For in the Kantian doctrine, symbolisation represents a degraded form of schematisation appropriate for Ideas (whether rational or aesthetic) that are in principle non-schematisable.

Now, Lautman precisely treats the categorial concept of symmetry as an Idea—perhaps not an Idea in the strict Kantian sense, but at the very least as a *concept of reflection*. Rather than make of it a 'maxim of physical judgment' (like the principle of relativity or the principle of least action), he makes it a concept at once constitutive and heuristic, as if he did not regard as pertinent the Kantian difference between determining judgment and reflective judgment—and therefore that between schematism and symbolisation.

Conclusion

There is obviously much that could be added on the importance Lautman's thought might have for the current state of epistemology, were its importance judiciously evaluated. Let us just briefly indicate some possibilities.

As eminent epistemologists such as Gaston Bachelard and Ludovico Geymonat have emphasised, in order correctly to think the movement of modern objective sciences, it is necessary to articulate a critical rationalism with a scientific historicism. This 'dialectic of historical status and objective

84. Ibid., 241.

85. Ibid., 254.

86. Ibid.

87. Cf. Petitot, 'Mathématique et Ontologie'.

88. Cf. Petitot, 'À propos de « Logos et Théorie des Catastrophes »' and *Morphogenèse du Sens II.*

status'[89] can nonetheless concern either the techno-experimental apparatuses of the sciences, or their eidetico-constitutive a priori. In the first case, the historicity will be that of (materialist) technicist practice, in the second case it will be that of an ideal (transcendental) dialectic. In his analyses of contemporary sciences as 'applied rationalism' and 'technical materialism', Bachelard opted for the first path.[90] While insisting upon the constitutive function of mathematics in the techno-rational 'ontogeneses' of physics, and founding the history of sciences upon a 'dialectic of epistemological obstacles and epistemological acts',[91] he subordinated the latter to the instrumental evolution of the disciplines in question. What is specific to Lautman is his having succeeded in thinking another dimension, an ontological dimension close to Heideggerian 'historiality': that of the historicity of the rational reconstruction of the real.

Another of Lautman's great achievements is to have unified Hilberto-Bourbakian axiomatic structuralism with a realist rationalism close on many points to that of Weyl, Planck, Einstein, or Heisenberg. This allowed him to go beyond the opposition between what Husserl, in a letter to Hermann Weyl, called the 'structural lawfulness' of nature and its 'specifically causal lawfulness'.[92]

However this may be, *structural* explanation and *causal* explanation (through entities not immediately given, invisible entities) come together in the concept of Weltbild (world-image), 'a neologism introduced by Planck to designate the schema of structure which is at the basis of a theory'[93] and a fundamental concept of Einstein's philosophy:

> Man tries to make for himself in the fashion that suits him best a simplified and intelligible picture of the world; he then tries to some extent to substitute this cosmos of his for the world of experience, and thus to overcome it. This is what the painter, the poet, the speculative philosopher, and the natural scientists do, each in his own fashion. Each makes this cosmos and its construction the pivot of his emotional life, in order to find in this way peace and security which he can not find in the narrow whirlpool of personal experience.[94]

As 'abstraction of the real through schematisation', a *Weltbild* must not be conflated with a direct 'axiomatisation' of empirical data. It is, rather, a hidden structure including 'a content exceeding its empirical basis and whose logical explication constitutes the discovery, which in return must be submitted to the control of experience'.[95] Like mathematical schematism, which its interpretation endows with a causal pertinence, it is a 'mixture' participating in the general project of all objective science, which is to reduce irrational empirical material to a formal theoretical rationality. For positivists, logical empiricists, and conventionalists, it is a question of nothing but an artefact with no ontological import: 'Poincaré's conventionalism, Duhem's nominalism, Mach's neo-positivism [...] have in common that they deny the physical *Weltbild* all ontological relation with the real.'[96] But in fact, in reducing the theory to being only an intellectually expedient systematisation of the facts, these anti-theoretical points of view remain deliberately on the surface of things and neglect the truth that, as Einstein affirms, 'science is the attempt to make correspond the chaotic diversity of our sensible experience with a logically uniform system of thought.'[97]

Now in their historical evolution, these *Weltbilder* realise a 'dialectic'. As 'schematic conformity with the real', a Weltbild is always partial and approximative. It is always engaged in a 'dialectical process that permits the improvement of the schematisation of the real.'[98] It thus does indeed constitute, as Gonseth conceives it, the deepening of the 'symbolisation'

89. L. Geymonat, *Lineamenti di filosofia della Scienza* (Milan: Mondadori, 1985), 128.

90. Cf. for example G. Bachelard, *L'Activité rationaliste de la physique contemporaine* (Paris: PUF, 1951).

91. Ibid., 36.

92. Letter of 9 April 1922. T. Tonietti, 'Quattro letttere di Edmund Husserl ad Hermann Weyl', in *E. Husserl e la crisi delle scienze europee* (University of Lecce, 1984).

93. P. Wehrlé, *L'Univers aléatoire* (Paris: Edition du Griffon-Vrin, 1956), 13. This little-known work, with a preface by Gonseth, is a good complement to his oeuvre. I thank my friend Guy le Gaufey for having brought it to my attention.

94. A. Einstein, paper given in 1918 in honour of Planck's birthday. Cited in J. Stachel, 'Einstein and the Quantum: Fifty Years of Struggle', in R. Colodny (ed), *From Quarks to Quasars* (University of Pittsburgh, 1986).

95. Ibid, 57.

96. Ibid. 30. It is a question of the standard interpretation but the error of conventionalism (cf. 2. a).

97. Cf. Stachel, 'Einstein and the Quantum'.

98. Ibid., 47.

upon which Lautman founded his conception of the relations between mathematics and reality.

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As enigmatic as it might be, the intelligibility of the universe is a fact. A fact we must somehow account for. In our view, Lautman was fundamentally in the right when, in order to do this, he adopted an anti-empiricist point of view and came back to the critico-phenomenological tradition. For in truth, there are indeed transcendental structures of objectivity that anticipate a priori the structure of empirical phenomena (ontological difference, comprehension, and genesis). The whole difficulty lies in achieving a mathematical conception of the a priori, something neither Kant nor Husserl managed to do for lack of an appropriate mathematical philosophy. Mathematization permits us to constrain the anticipation within its form and to diversify it in its consequences. It thereby allows us to test it.

As far as we are concerned, the critique we address to Lautman is certainly not that of being an idealist. Quite on the contrary. Rather, we criticise him only for not always maintaining a good Kantian distance between schematisation and symbolisation, between determinative judgment and reflective judgment, between categories and Ideas, between Analytic and Dialectic, in short between knowledge and thought. This indecision relative to the irreversible gains made by the critical method, let us repeat, results in a general difficulty in grasping quite what it is that prevents Lautman's dialectic, which would be Platonist, from surreptitiously becoming a Hegelian dialectic limited to mathematics.

Such a critique is in accord with Cavaillès's critique of intuitionism, which, according to him, 'conflates the dialectical moment of the positing of the concept and the transcendental moment of its schematisation'.[99]

But it does not at all invalidate the remarkable pertinence of Lautman's research programme. It is incomprehensible and unjust that such an inspired mind has been so little celebrated. For there is truly a genius, an intellectual and a spiritual genius, in dedicating oneself by vocation to the patient study of 'this increate germ which contains in it at once the elements of a logical deduction and of an ontological genesis of sensible becoming'.[100] Yes, it remains to remake the *Timaeus*.

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99. J. Cavaillès, 'Transfini et continu' [1941], in *Philosophie Mathématique* (Paris: Hermann, 1962), 253-274: 272.

100. Lautman, *Essai*, 255.